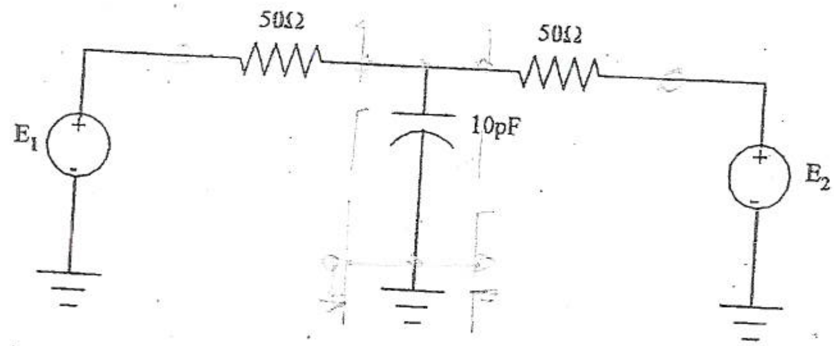
1. For the circuit shown below, the signal frequency is 1 GHz. Find the augmented short-circuit admittance matrix $\underset{\sim}{Ya}$ and the scattering matrix $\underset{\sim}{S}$.



Solution

$$\underset{\sim}{Z} = \frac{1}{sC}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \underset{\sim}{Z_a} = \underset{\sim}{R} + \underset{\sim}{Z} = \begin{bmatrix} R+Z & Z \\ Z & Z+R \end{bmatrix}$$

$$\underset{\sim}{Y_a} = \frac{1}{R^2+2RZ}\begin{bmatrix} Z+R & -Z \\ -Z & Z+R \end{bmatrix}$$

$$\underset{\sim}{Y_{an}} = \frac{R}{R^2+2Z}\begin{bmatrix} Z+R & -Z \\ -Z & Z+R \end{bmatrix}$$

$$\underset{\sim}{S} = \underset{\sim}{I} - 2\underset{\sim}{Y_{an}} = \frac{1}{R+2Z}\begin{bmatrix} (R+2Z)-2(R+Z) & 0+2Z \\ 2Z & -R \end{bmatrix}$$

$$\underset{\sim}{S} = \frac{1}{R+2/j\omega C}\begin{bmatrix} -R & 2/j\omega C \\ 2/j\omega C & -R \end{bmatrix}$$

For $R = 50\,\Omega$, $C = 10\,pF$, $\omega = 2\pi 10^9$ rad/s

$$2/j\omega C = -j/(\pi \cdot 10^9 \cdot 10^{-11}) \cong -j\,31.83\,\Omega$$

$$\underset{\sim}{S} \cong \frac{-1}{50-j31.83}\begin{bmatrix} 50 & j31.83 \\ j31.83 & 50 \end{bmatrix}$$

TABLE 4.2 Conversions Between Two-Port Network Parameters

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\dfrac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\dfrac{2Z_{12}Z_0}{\Delta Z}$	$\dfrac{-2Y_{12}Y_0}{\Delta Y}$	$\dfrac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\dfrac{2Z_{21}Z_0}{\Delta Z}$	$\dfrac{-2Y_{21}Y_0}{\Delta Y}$	$\dfrac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\dfrac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\dfrac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\dfrac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\dfrac{Y_{22}}{\|Y\|}$	$\dfrac{A}{C}$
Z_{12}	$Z_0\dfrac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\dfrac{-Y_{12}}{\|Y\|}$	$\dfrac{AD-BC}{C}$
Z_{21}	$Z_0\dfrac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\dfrac{-Y_{21}}{\|Y\|}$	$\dfrac{1}{C}$
Z_{22}	$Z_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\dfrac{Y_{11}}{\|Y\|}$	$\dfrac{D}{C}$
Y_{11}	$Y_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{22}}{\|Z\|}$	Y_{11}	$\dfrac{D}{B}$
Y_{12}	$Y_0\dfrac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{12}}{\|Z\|}$	Y_{12}	$\dfrac{BC-AD}{B}$
Y_{21}	$Y_0\dfrac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-Z_{21}}{\|Z\|}$	Y_{21}	$\dfrac{-1}{B}$
Y_{22}	$Y_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{Z_{11}}{\|Z\|}$	Y_{22}	$\dfrac{A}{B}$
A	$\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{11}}{Z_{21}}$	$\dfrac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\dfrac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{\|Z\|}{Z_{21}}$	$\dfrac{-1}{Y_{21}}$	B
C	$\dfrac{1}{Z_0}\dfrac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{1}{Z_{21}}$	$\dfrac{-\|Y\|}{Y_{21}}$	C
D	$\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{Z_{22}}{Z_{21}}$	$\dfrac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$.

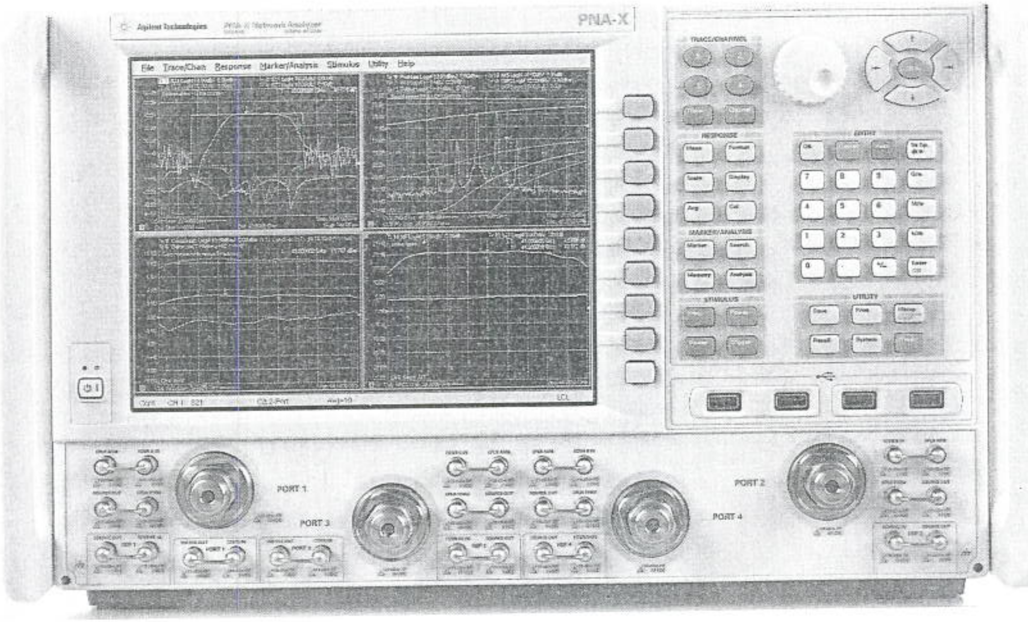


FIGURE 4.7 Photograph of the Agilent N5247A Programmable Network Analyzer. This instrument is used to measure the scattering parameters of RF and microwave networks from 10 MHz to 67 GHz. The instrument is programmable, performs error correction, and has a wide variety of display formats and data conversions.

Courtesy of Agilent Technologies.

waves on all ports except the jth port are set to zero, which means that all ports should be terminated in matched loads to avoid reflections. Thus, S_{ii} is the reflection coefficient seen looking into port i when all other ports are terminated in matched loads, and S_{ij} is the transmission coefficient from port j to port i when all other ports are terminated in matched loads.

EXAMPLE 4.4 EVALUATION OF SCATTERING PARAMETERS

Find the scattering parameters of the 3 dB attenuator circuit shown in Figure 4.8.

Solution

From (4.41), S_{11} can be found as the reflection coefficient seen at port 1 when port 2 is terminated in a matched load ($Z_0 = 50\ \Omega$):

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma^{(1)}|_{V_2^+=0} = \left.\frac{Z_{\text{in}}^{(1)} - Z_0}{Z_{\text{in}}^{(1)} + Z_0}\right|_{Z_0 \text{ on port 2}},$$

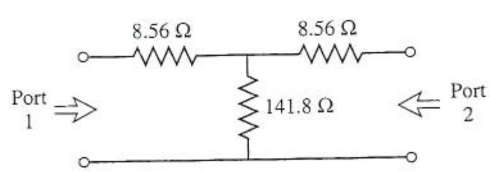


FIGURE 4.8 A matched 3 dB attenuator with a 50 Ω characteristic impedance (Example 4.4).